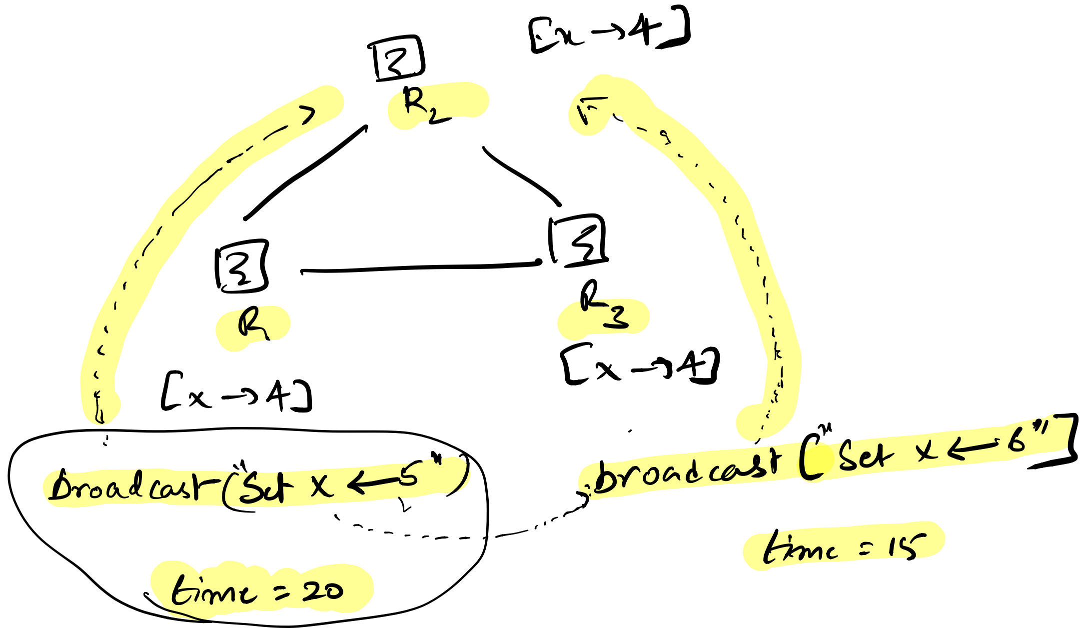
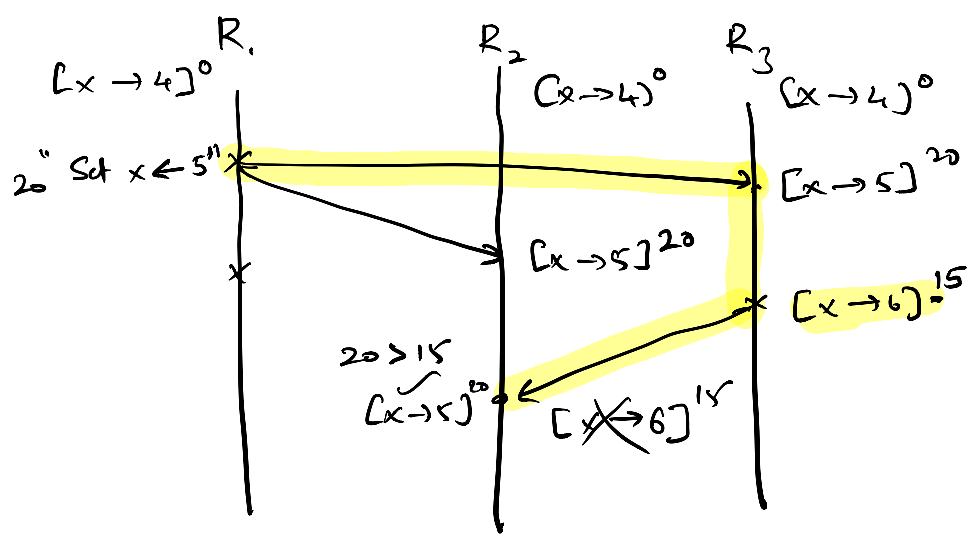
Welcome to CSCI 7000-001 Lec 3! (Jan 21)

Recap : * Asynchronous Model

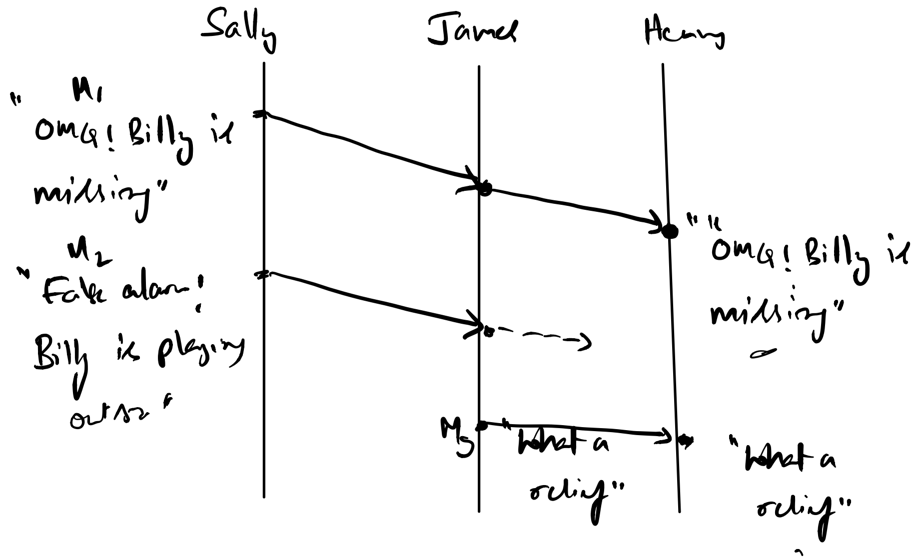
* Causality

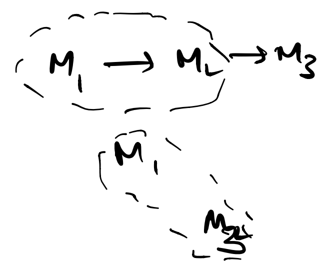




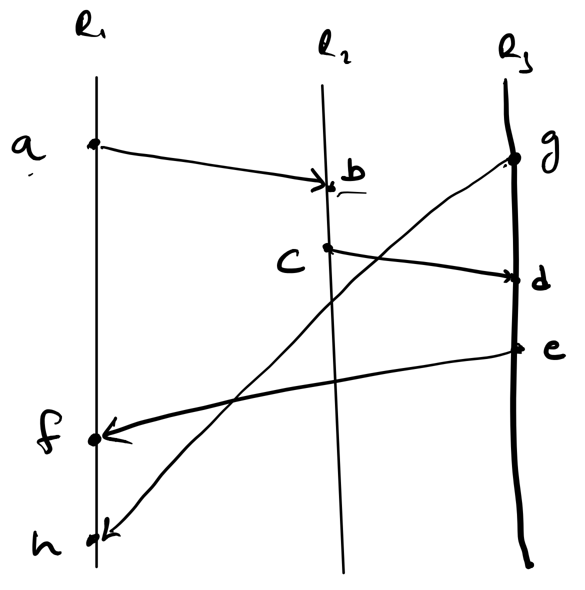
(x ← 5) $\xrightarrow{hb}$ (x ← 6)

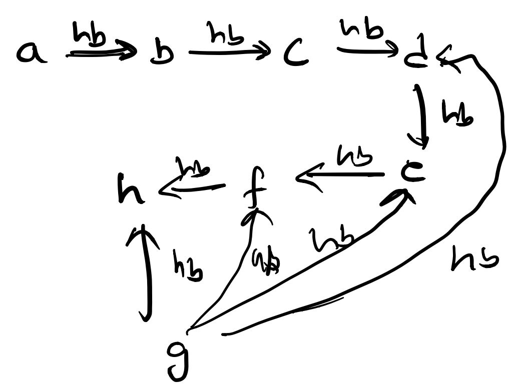
[Bailis et al's SIGMOD 13]

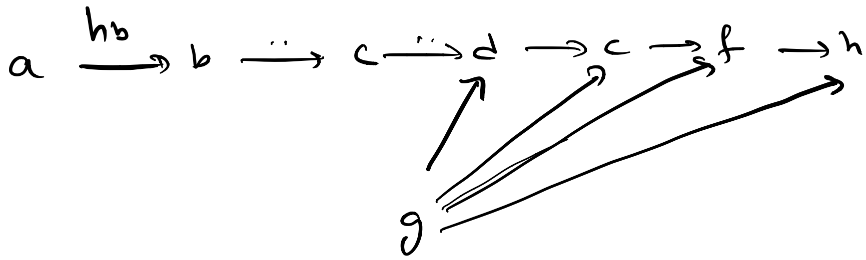




James msg causally depends on both of Sally's messages.







* How are $\{a, b, c\}$ & g related?

They are not causally related.

→ Happens Before Relation (hb)

E : Set of all events in a distributed Execution

Strict Partial order

(1) $hb \subseteq E \times E$

(2) Transitivity: $\forall (e_1, e_2, e_3 \in E)$

$hb(e_1, e_2) \wedge hb(e_2, e_3) \Rightarrow hb(e_1, e_3)$

(3) Anti-symmetric:

$\forall (e_1, e_2 \in E).\ hb(e_1, e_2) \Rightarrow \neg hb(e_2, e_1)$

(4) Irreflexive:

$\forall (e \in E).\ \neg hb(e, e)$

$(<)$ Strict Total Order

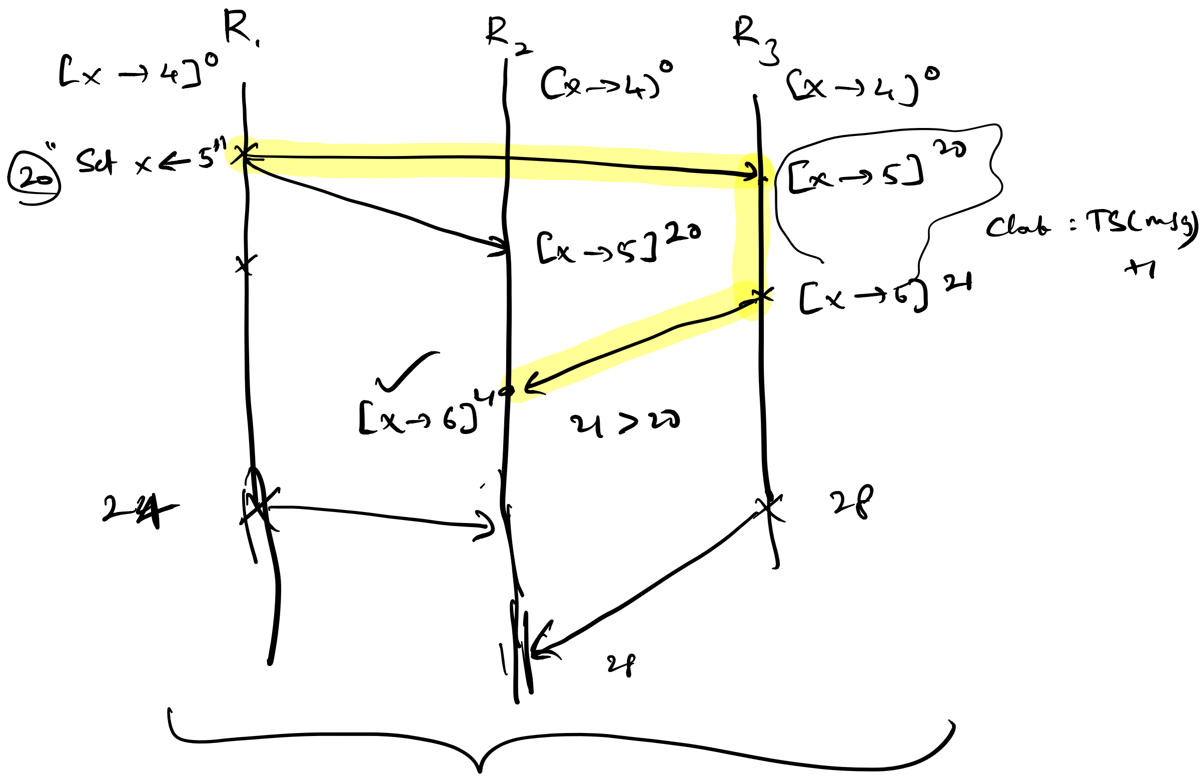
(1) $(<) \subseteq N \times N$

(2) Transitive

(3) Anti-symmetric

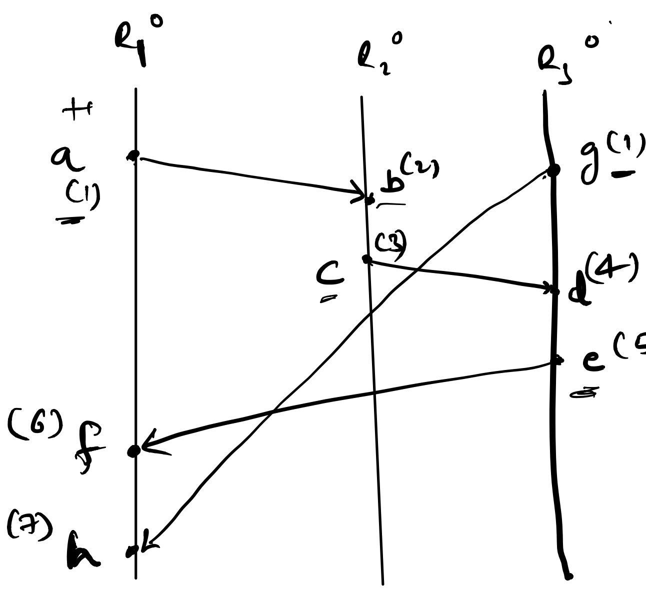
(4) Irreflexive.

(5) $\underbrace{\forall n_1, n_2.}_{\text{Totality}} \; (n_1 < n_2 \lor n_2 < n_1)$



Lamport timestamps

my_clock

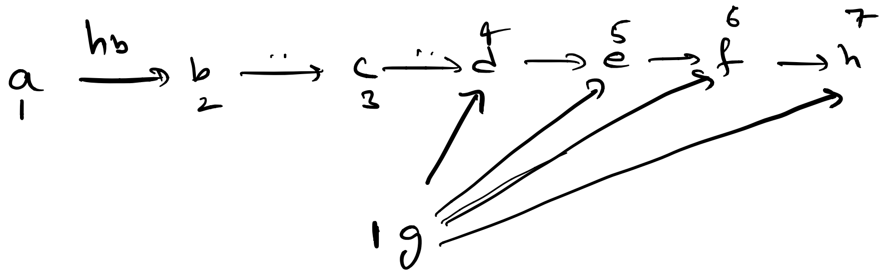


on receive. Msg

my-clock := Max (my-clock, msg.ts) +1

on send:

my-time := my-time +1



$$\forall e_1, e_2.\ e_1 \xrightarrow{hb} e_2 \Rightarrow \underbrace{ts(e_1) < ts(e_2)}$$

$$P \Rightarrow Q$$

$$\neg Q \Rightarrow \neg P$$

$$\neg(ts(e_1) < ts(e_2)) \Rightarrow \neg(e_1 \xrightarrow{hb} e_2)$$

→ Lamport timestamps are consistent with causal ordering. But they do not fully characterize

Causal ordering

→ <u>Next Step</u>: Timestamping scheme that fully characterizes causal ordering

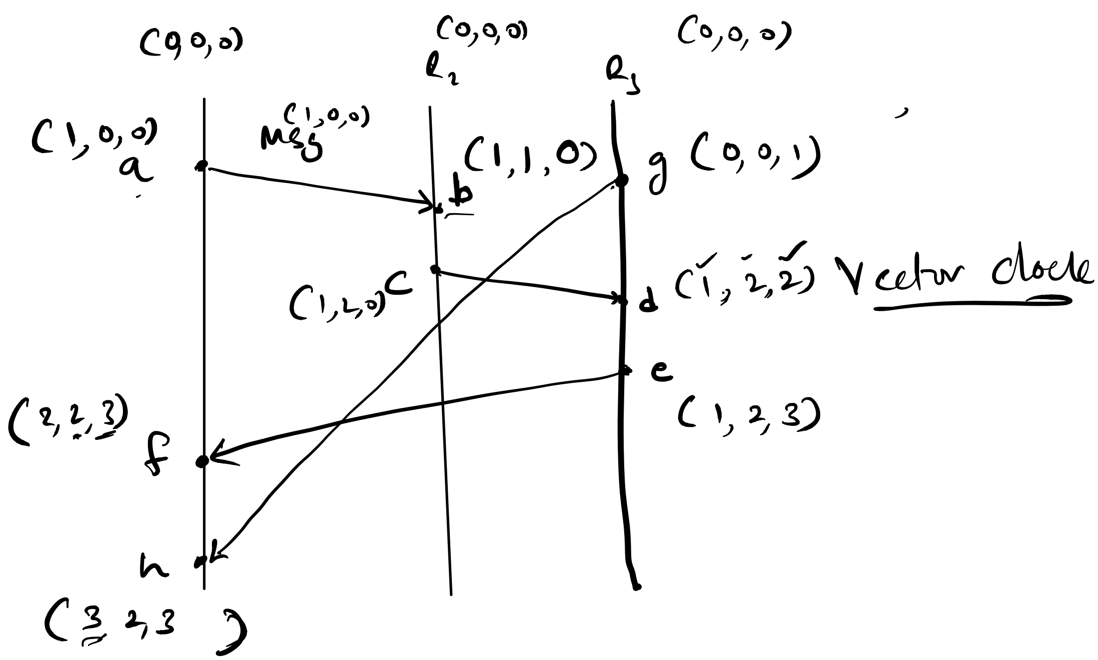
→ $< \subseteq \mathbb{N} \times \mathbb{N}$ is a total order

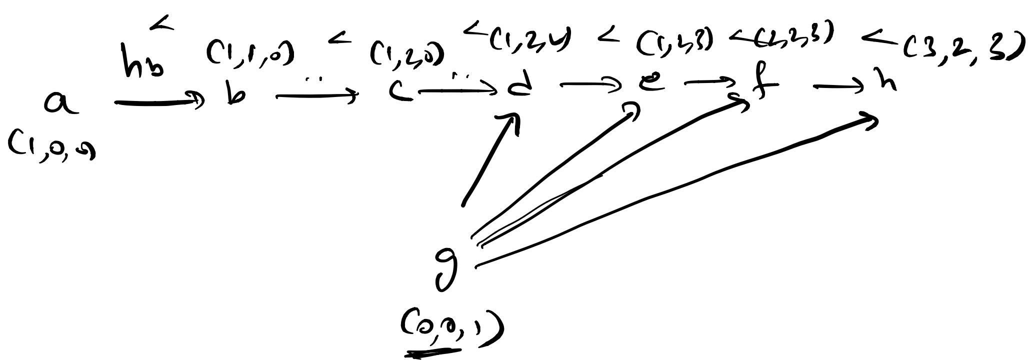
$$(x_1, y_1) < (x_2, y_2) \iff x_1 < x_2 \wedge y_1 < y_2$$

$$\underbrace{< \;\subseteq\; (\mathbb{N}, \mathbb{N}) \times (\mathbb{N}, \mathbb{N})}_{\text{Partial order}}$$

$(3, 4) \qquad (1, 5)$

→ Vectors of Natural Numbers.





(1,0,0) < (1,1,0) < (1,2,0) < . . .

$\overset{g}{(0,0,1)}$ | $\overset{a}{(1,0,0)}$, $\overset{b}{(1,1,0)}$, $\overset{c}{(1,2,0)}$

→ Vector clocks completely characterize causal orders.